# MULTIPLE TRANSONIC SOLUTIONS AND A NEW CLASS OF SHOCK TRANSITIONS IN SOLAR AND STELLAR WINDS

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ABSTRACT

The steady isothermal solar wind equations are shown to admit, under certain circumstances, multiple transonic solutions when, for example, momentum deposition gives rise to multiple critical points in the flow. These multiple solutions consist of a continuous solution and solutions which involve shock transitions between critical solutions. The ambiguity arising from the multiplicity of the solutions can be resolved by following the time evolution of a wind profile with one critical point. Results of the numerical integration of the time-dependent equations with momentum addition show that each of these multiple solutions is physically accessible and depends on the rate of change of momentum deposition. These results suggest that standing shocks are likely to be present in the inner solar wind flow.

## 1. Introduction

The importance of multiple critical points arising in the solar wind flow as a result of momentum addition, heat addition, and/or area divergence was first pointed out by Holzer [1977]. Leer and Holzer [1980] noted that if the wind velocity becomes supersonic at an inner critical point, shifting from one further downstream, the requirement for extended energy addition originating from the coronal base to the supersonic flow could be relaxed (e.g. waves need not propagate far out in the corona to be dissipated and add energy to the supersonic flow). In this paper we point out additional interesting properties of the transonic wind solutions to the isothermal solar wind equation of motion when multiple critical points arise in the flow. We show that, if as a result of momentum addition or rapid area divergence the wind velocity becomes supersonic at an inner critical point, leaving other critical points further downstream, the steady solar wind equation of motion can admit multiple transonic solutions, one of which is continuous, while the others involve a shock transition. We also show that the transonic wind solution with initially one critical point can evolve in time to either one of these multiple solutions depending on the rate of change of momentum deposition to the flow.

## 2. Mathematical Description

To study the time evolution of an isothermal, one-fluid proton-electron solar wind with momentum addition we write the time-dependent mass and momentum conservation equations for a locally radial flow in the form,

$$\frac{\partial n}{\partial t} + \frac{1}{A}\frac{d}{dr}( nAv) = 0 \tag{1}$$

and

$$\frac{\partial}{\partial t}(nv) + \frac{1}{A}\frac{d}{dr}( nAv^2) = -\frac{1}{m}\frac{dP}{dr} - \frac{GM}{r^2} + nD \tag{2}$$

where A is the cross-sectional area of a flow-tube, n is the electron (or proton) density, m is the proton mass, and p = 2nkT is the fluid pressure. The phenomenological term D has the dimensions of force per unit mass, and nmD represents the volume rate of momentum addition [see Holzer [1977]]. To discuss the solutions of the time-dependent equations in connection with those of the steady state equations we combine the above two equations in the form,

$$\left(\frac{1}{c^2}\frac{\partial v}{\partial t} - \frac{1}{nv}\frac{\partial n}{\partial t}\right) + \frac{M^2 - 1}{2M^2}\frac{dM^2}{dr} = \frac{2}{r} - \frac{GM}{r_s c^2 r^2} + \frac{d \ln f}{dr} + \frac{D}{c^2} \tag{3}$$

where M = v/c is the flow Mach number, and c the constant sound speed. The cross-sectional area of a flow-tube is written as $A(r) = A_s(r/r_s^2)f(r)$, with $r_s$ the coronal base, and f(r) a function parameterizing the divergence from spherical symmetry. The properties of the solution topologies of the steady state limit ($\partial/\partial t = 0$) are discussed in detail in Habbal and Tsinganos [1983]. We merely note here the equivalent effects of rapid area divergence (d ln f/dr) and momentum addition ($D/c^2$) in producing topological changes in the solutions, and the possibility for steady shock transitions to occur between critical solutions as determined by the condition

$$v_1 v_2 = c^2 \tag{4}$$

## 3. Results

The changes in the steady transonic wind solution as a result of changes, f(r), in the areal divergence of the flow tube, where [see Kopp and Holzer [1976]]

$$f(r) = \frac{f_{max}\, e^{(r-r_1)/\sigma} + 1 - (f_{max}-1)\, e^{(r_s-r_1)/\sigma}}{e^{(r-r_1)/\sigma} + 1} \tag{5}$$

are shown as thick solid lines in Figure 1 for different values of $f_{max}$. Two multiple transonic solutions coexist as soon as the velocity becomes supersonic at an inner critical point, Figure 1d. For a further increase in the areal divergence of the flow tube, three solutions are possible and consist of one continuous transonic solution and two transonic solutions involving a steady shock transition between critical solutions, Figure 1e-1f.

The temporal behavior of the wind profile with initially one critical point to a final steady profile with multiple critical points as a result of momentum addition is shown in Figures 2 and 3. In Figure 2, the strength, $D_0$, is such that the steady state equations allow for multiple solutions to coexist, such as in Figure 1e. The resulting steady state velocity and density solutions are shown in the top panels, Figure 2a-2b, while the time evolution profiles are shown below. To vary the rate of momentum addition, we introduce the parameter $\tau$, where

$$D(r,t) = D_0(1 - e^{-t/\tau})\, e^{-((r-r_p)/\alpha)^2} \tag{6}$$

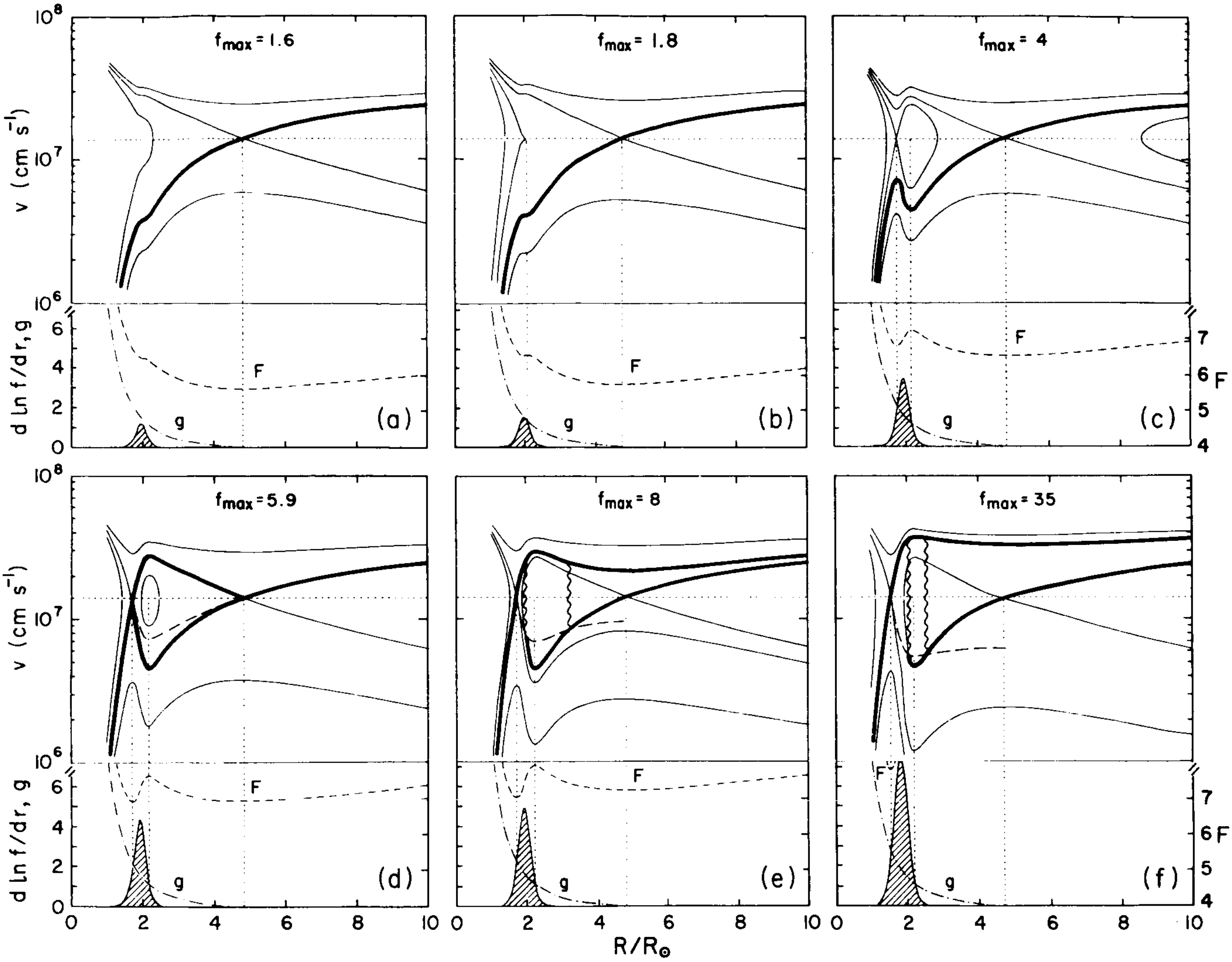


Figure 1. Sequence of solution topologies resulting from an increase in the area divergence of the flow tube, as indicated by $f_{max}$, for $r_1 = 2\ r_s$ and $\sigma = 0.1\ r_s$ [see (5)]. Additional critical points arise in the flow as the area function, dlnf/dr, intersects the curve $g = GM_s c^2 r^2 - 2/r$. The transonic wind solutions are drawn as thick solid lines, and the shock transitions, when present, are indicated by the wavy lines satisfying the condition (4). The wind is assumed isothermal at $T = 1.2 \times 10^6$ K with a sonic point at $4.8\ r_s$ for a spherically symmetric flow [from Habbal and Tsinganos [1983]].

The results shown correspond to three different values of $\tau$ . The integration time step is 10 s, and the time interval between neighboring curves is $10^4$ s. The initial state is the solution to the steady state solar wind equation of motion with no momentum addition for an isothermal wind at $T = 1.2 \times 10^6$ K, with a sonic point at $4.8\ r_s$. The momentum is then applied in time either slowly (large $\tau$), or rapidly (small $\tau$ ). For $\tau = 4 \times 10^4$ s, Figure 2c-2d, the velocity and density profiles evolve to form a standing shock at the inner position predicted by the steady state calculations, while for $\tau = 2 \times 10^4$ s, Figure 2e-2f, the velocity and density profiles evolve with a standing shock further downstream

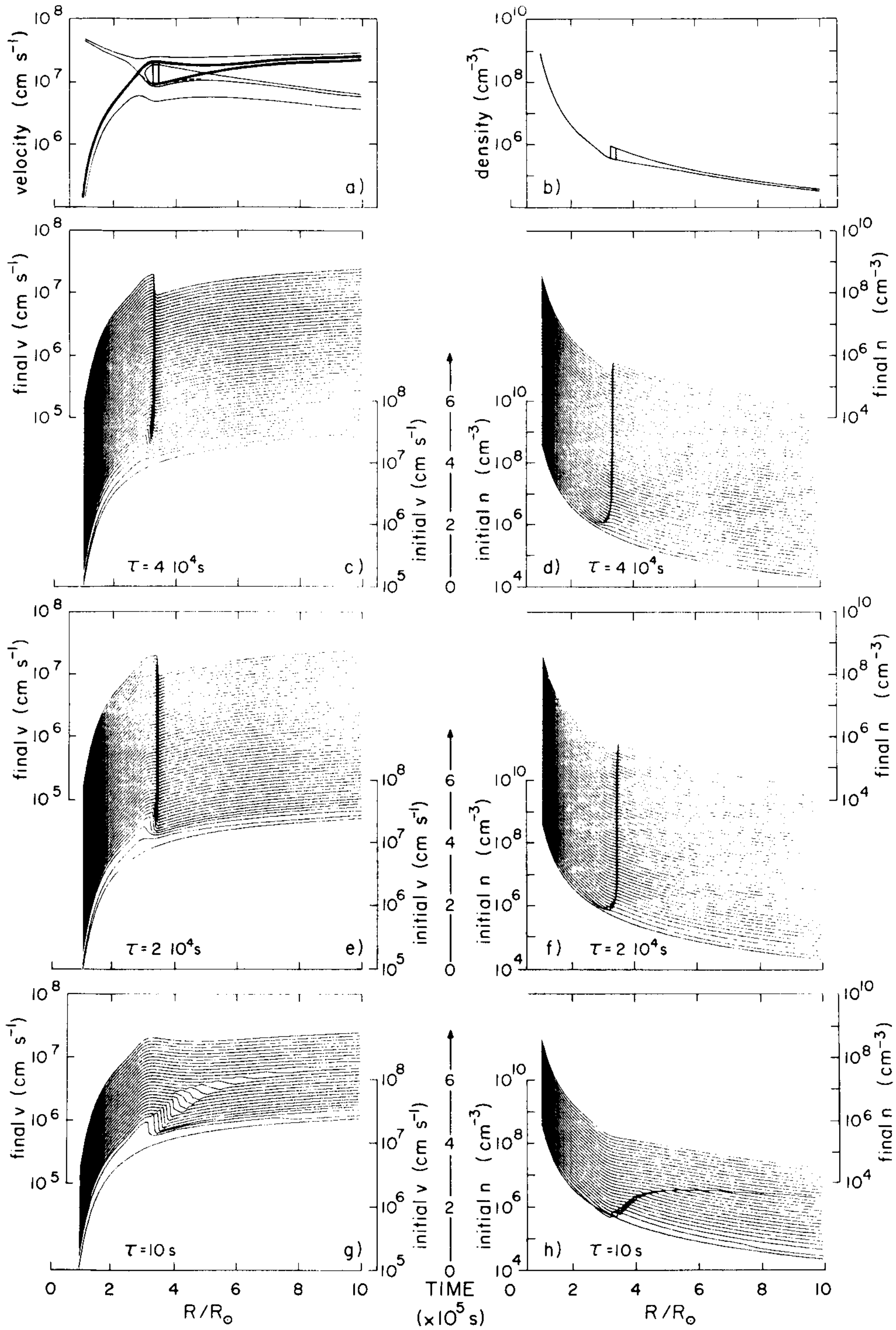


Figure 2. Sequence of time evolution of the velocity and density profiles to a final steady state as a result of momentum addition, with $D_0 = 2.7\times10^3$, $\alpha = 0.3$, and $r_p = 3r_s$, for different values of $\tau$ [see (6)]. The sequence shows how each of the steady multiple transonic solutions, shown as dark solid lines in (a)-(b), are accessible. In (c)-(d), the wind profile evolves in time to the steady state solution with the standing shock at the inner position for $\tau = 4\times10^4$ s. The second standing shock develops for a slightly smaller value of $\tau = 2\times10^4$ s, (e)-(f). For $\tau = 10$ s, the velocity and density profiles evolve to continuous ones.

than in the previous case. By reducing $\tau$ further to $\tau$ = 10 s, the velocity and density profiles evolve in such a way that a shock discontinuity develops only temporarily, as soon as the velocity becomes sonic closer to the base, but disperses and propagates outwards. Hence, by changing the rate of momentum deposition, this example illustrates how each of the multiple solutions is physically realizable. For a further increase in $D_0$ (Figure 3), the condition for the existence of steady shocks is no longer satisfied. The velocity and density profiles evolve to continuous steady solutions for either value of $\tau$, despite the temporary formation of a shock discontinuity.

## 4. Conclusion

In the present study we have illustrated how the multiplicity of the transonic solutions to the steady isothermal wind equations arises as a result of changes in the areal divergence of the flow tube and/or as a result of momentum addition. By changing the rate of momentum deposition, and by following the corresponding temporal evolution pattern of the wind profile with initially one critical point, we have shown how each one of the multiple solutions is physically realizable and corresponds to different intermediate temporal states of the solar wind. In particular, we have shown how a transonic solution with a standing shock could occur in the wind flow as predicted by Holzer [1977] (albeit without the restriction of an isothermal flow).

The multiplicity of the transonic solutions to the steady isothermal solar wind equations, however, is not unique to this study. Multiple solutions have originally been found by Leer et al. [1982] in their study of Alfvén wave driven stellar winds. They find two continuous transonic solutions characterized by different velocities at the base, for a certain range of wave damping lengths. In our case, the steady multiple transonic solutions exhibit some form of degeneracy since they all have the same velocity at the base, although the continuous solution and those involving a shock transition have different asymptotic flow speeds.

Finally, as an example based on our results, we note that if the rate of change of momentum addition or the temporal change in the area divergence of a flow tube, such as in a coronal hole, occurs over time scales of the order of $10^4$ s, and the final value reached is maintained for time scales of the order of $10^5$ s, i.e. days, a standing shock can develop within 5 $r_s$ from the coronal base for a solar wind at say $1.2\times10^6$ K. In a realistic solar wind model the formation of a standing shock in the solar wind could lead to a reduction in the energy flux per particle, since the temperature rise in the shock will result in energy loss by thermal conduction to the base if the shock occurs within 1 or 2 $R_s$ of the inner critical point [E. Leer, private communicaton [1983], see also Holzer and Leer [1980] for a detailed discussion of conductive solar wind models]. For an isothermal wind, however, and an isothermal shock, although the infinite thermal conductivity is implicit, the loss of energy flux to the wind as a result of shock formation is apparent in the lower asymptotic flow speed, yet there is no corresponding increase in particle flux at the base when compared with the particle flux associated with the continuous transonic solution.

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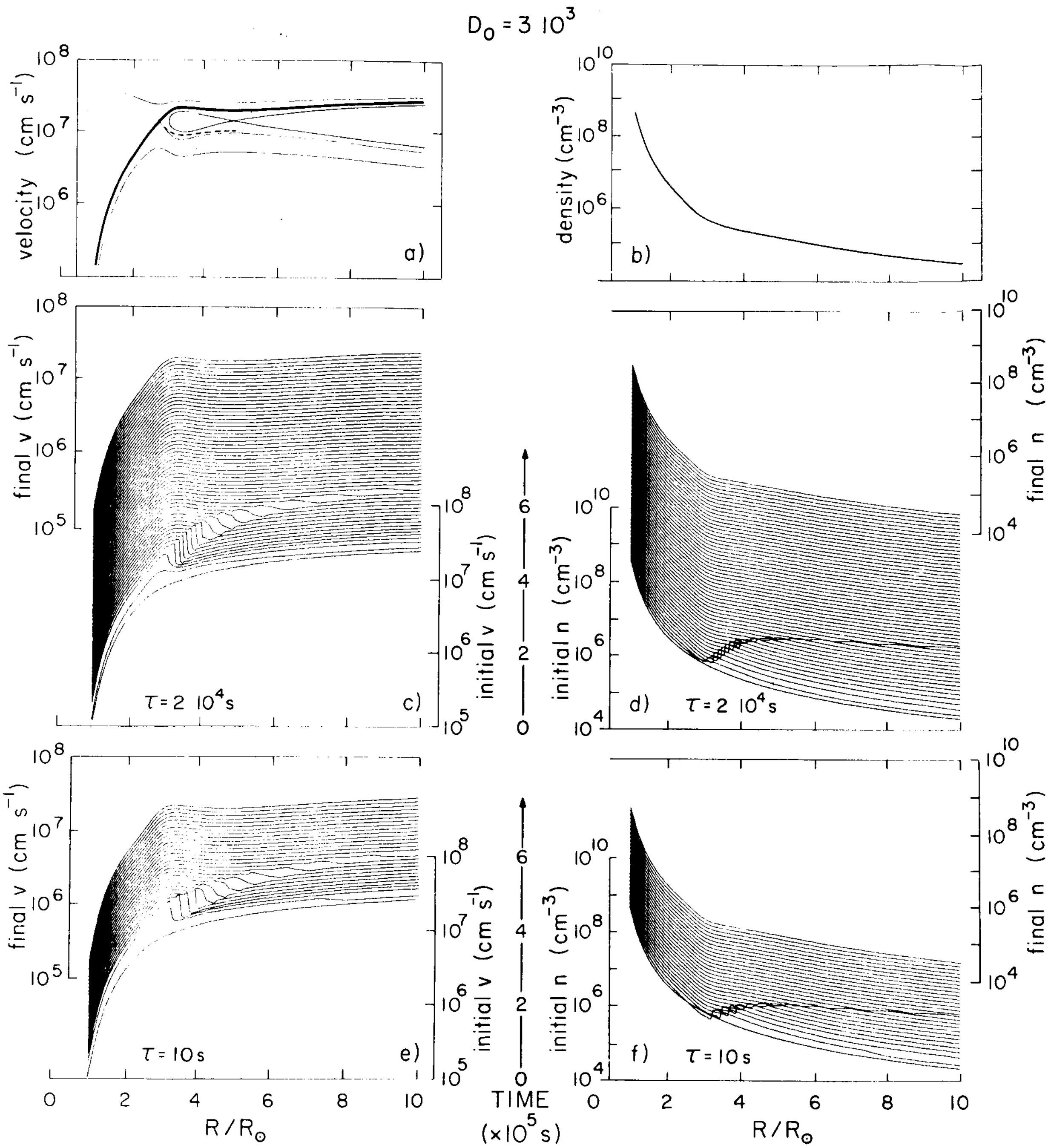


Figure 3. Same as Figure 2 for a strength of momentum addition such that the steady state equations do not admit multiple solutions. The temporal behavior of the wind profile is such that shock discontinuities form temporarily in the flow but disperse and propagate outwards.

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